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Prediction of abrasive particle velocity in a high pressure water jet and effect of air on acceleration process

A. TAZIBT *, **, N. ABRIAK * and F. PARSY **

ABSTRACT. – In this paper, a theoretical study of a particle velocity in a high speed water jet flowing within a mixing tube is presented. It is based on an analytical method of solution of a differential non-linear equation of particle motion which includes two unknowns: the velocities of the particle and of the water jet. The flow is assumed to be one-dimensional and the particle is isolated along the axis of the jet where the conservation of momentum is essential for analysis. As a result, we have obtained two simple analytical equations which represent, respectively, the variations of the velocities of the particle and of the water jet as function of distance along the mixing tube. The present solution takes account of the several forces (e.g.: drag, virtual mass force) that act on the particle and also elucidates the effect of air on the acceleration process. Air is entrained in the real conditions of abrasive formation. Comparison of results obtained by the present modelling with those found in the literature shows a good agreement. In one particular test application of such an abrasive water jet in cutting shows that the experimental depths of cut are near those estimated using the theoretical particle velocity.

Nomenclature

A_a	interfacial force per unit length of mixing tube
C_d	drag coefficient
d	mixing tube diameter
D	nozzle diameter
D_a	particle diameter
$\dot{E}c$	particle kinetic power at impact
$\dot{E}r$	cutting power
f_d	drag force per unit volume of particle
f_{vm}	virtual mass force per unit volume of particle
H	depth of cut
m_a	particle mass
$\dot{m}_a$	abrasive mass flow rate
$\dot{m}_w$	water mass flow rate
$\dot{m}_r$	removal mass flow rate of cut material
P	hydraulic pressure of water
u	traverse rate of cutting
V	volume of abrasive particle

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V_a	abrasive particle velocity
V_{ao}	initial abrasive particle velocity
V_{ac}	abrasive particle velocity at impact
V_{eq}	equilibrium velocity
V_w	water velocity
V_{wo}	initial water velocity
X	distance of impact
x_k	location of phase-k
α	volume fraction of abrasives
β	volume fraction of void
ε	specific energy of cut
ρ	density of cut material
ρ_a	density of abrasive
ρ_{fl}	density of fluid (droplet and air)
ρ_g	density of air
ρ_w	density of water
Ω	cross-sectional area of jet
Ω_a	cross-sectional area of particle

1. Introduction

Cold cutting using a high speed abrasive waterjet is now widely used in industrial applications because of the ability of the system to cut materials without modifying their mechanical properties. This aspect becomes vital when cutting such materials as plastics, leather and composites. In addition, the hardest materials can be cut more economically than by conventional systems. The system is finding a wide field of application in, for example, aeronautical and civil engineering, and in the motor industry.

There are two types of abrasive waterjets in practical use. The first is called the Abrasive Slurry Jet (ASJ) (Fairhust *et al.,* 1986) where abrasive particles are premixed with water in a high pressure tank. The resulting mixture (slurry) is then directly pumped and accelerated in a long high pressure tube. The particular feature of such a system is that the mixture is composed of two phases: particles (solid) and water (liquid).

The second type is called the Abrasive Water Jet (AWJ) or PASER for Particle Stream Erosion where abrasives are sucked into the water jet by a venturi phenomenon, created by the high speed waterjet at the entrance of the mixing chamber. The particles are entrained and accelerated by the jet in a short carbide mixing tube. The resulting mixture is composed of three phases: particles (solid), water (liquid) and air (gas).

Difficulties arise with this cutting process in the control and optimisation of the working parameters such as the hydraulic pressure, mass flow rate of abrasives, nozzle diameter, velocity of abrasive particles, depth of cut, traverse rate and the characteristics of the material to be cut.

The complexity of the cutting mechanism makes theoretical modelling, which is able to take account of the great number of parameters involved, difficult. This difficulty is overcome by decoupling the process, considering the acceleration and curring aspects separately. These two processes must be coupled together by consideration of the particle velocity at impact.

The material is cut by abrasion due to the high kinetic energy of the particles. It is therefore of the first importance to determine the particle velocity. Hashish (1984) has developed a cutting model based on the Finnie and McFadden (1978) erosion theory, relating the cutting depth to the velocity. The particle velocity at impact is estimated from the momentum theorem, assuming that the mixing tube is long enough for equilibrium to be attained. In service the particle velocity is not known, so that the present theoretical model has been developed to provide an estimate at practical distances, for both AWJ and ASJ systems.

By an acceleration model we here mean a mathematical equation giving the evolution of the phase velocity along the mixing tube. Among the models relevant to this study, there is the complete one of Drew (1983) and those of Abudaka and Crofton (1989) and Nadeau *et al.* (1991).

In his modelling, Drew (1983) assumes that each material involved can be described as a continuum, governed by the partial differential equations of continuum mechanics. At the interphase, jump conditions express the conditions of conservation of mass and momentum. While this modelling seems more complete, analytical solutions for the differential equations of motion could not be found because the equations are non-linear.

Abudaka and Crofton (1989) write a classical equation of motion of a particle within the fluid flow along the mixing tube, taking account of the drag action, the water velocity being assumed to be constant. As a result, this model has two constants which must be set in an *ad hoc* fashion.

Nadeau *et al.* (1991) write the equations of the momentum transfer term at the solid-liquid interface. The waterjet and abrasive velocities are variable within the constraints of conservation of momentum. By the Runge-Kutta method, Nadeau *et al.* numerically determine the two solutions of the differential equation of motion; one for the abrasive particle and the other for water.

In the present work, a more general model which does not have these limitations and does not require an *ad hoc* adjustment of constants is derived.

2. Mathematical description

2.1. Assumptions

We first assume that the mixture is composed of two phases: solid phase (particles) and liquid (water). Air entrainment is initially neglected. The other basic hypotheses are listed below:

- the two phase flow is one-dimensional,
- momentum is conserved during the acceleration process,
- the particles are spherical and are flowing on the jet axis,
- the particles are sufficiently far apart as to be regarded as isolated in the stream,
- only drag forces act on the particles,
- wall friction, pressure gradient, viscosity and gravitational forces are neglected.

2.2. Description of mixing and acceleration mechanisms

As shown in Figure 1, high pressure water (200 to 400 MPa) enters the mixing chamber at high speed through a 0.3 mm diameter sapphire nozzle, and creates a depression. The abrasive particles (0.15 to 0.4 mm mean diameter) are entrained by air (venturi phenomenon) within a flexible tube from a hopper and enter the mixing chamber with a low velocity.

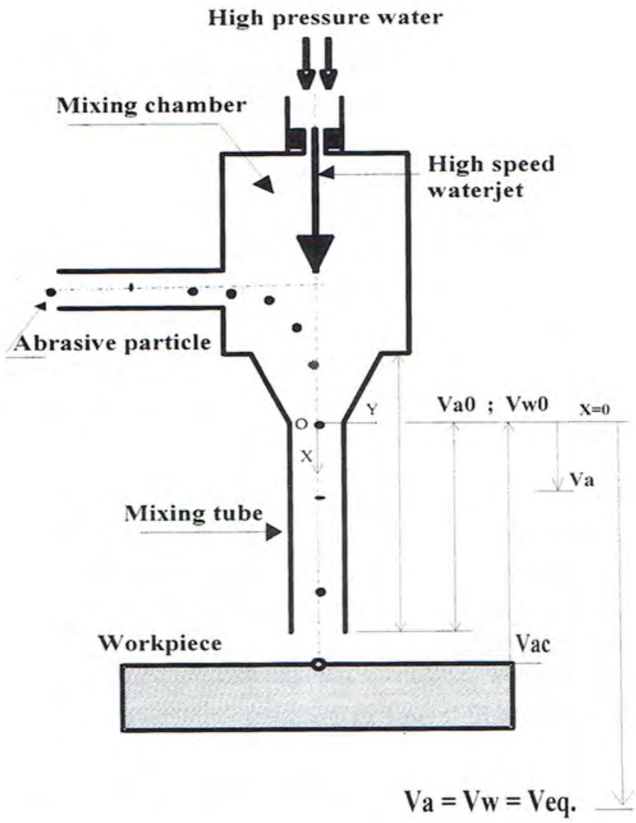

Fig. 1. – Schema of the Abrasive Water Jet cutting head and the mechanism of the mixing and acceleration process.

As the resulting mixture moves down the mixing tube (0.8 to 2 mm diameter and 7 cm long), momentum is transferred from the high velocity water to the low velocity particles. According to conservation of momentum, it is expected that the particle velocity will increase and the water velocity will decrease. If the mixing tube is sufficiently long, the particle and water velocities will come to be equal.

2.3. Spatial acceleration

Following Lahey *et al.* (1980), one can assume a string of non-interactiong particles flowing through a mixing tube. This assumption makes it possible for us to drop the temporal terms, since each particle in the string will have the same velocity when it reaches a particular axial location.

2.4. Equation of solid phase momentum

We assume that only drag forces act on the abrasive particles and, according to the assumptions stated in a previous section, the appropriate one-dimensional momentum equation for the accelerating particle within the waterjet is given by

$$\rho_a V_a \frac{dV_a}{dx} = -f_d \tag{1}$$

where

$$f_d = \frac{3}{4} C_d \frac{\rho_w}{D_a} (V_a - V_w) \mid V_a - V_w \mid . \tag{2}$$

As one can see, Eq. (1) is non-linear with two unknowns, V_a and V_w, which are, respectively, the particle and water velocity. The analytical solution of such an equation requires a second equation which must include the two unknowns.

Following Nadeau *et al.* (1991), we can write:

– *Solid phase (particle):*

$$\dot{m}_a \frac{dV_a}{dx} = A_a \tag{3}$$

where

$$\dot{m}_a = \alpha \rho_a \, \Omega \, V_a . \tag{4}$$

– *Liquid phase (water):*

$$\dot{m}_w \frac{dV_w}{dx} = -A_a \tag{5}$$

where

$$\dot{m}_w = (1 - \alpha) \rho_w \, \Omega \, V_w . \tag{6}$$

and where

$$A_a = -\frac{3}{4} \alpha \rho_w \, \Omega \frac{C_d}{D_a} (V_a - V_w) \mid V_a - V_w \mid . \tag{7}$$

A_a represents the interfacial force per unit of distance along the mixing tube, α the volumetric fraction of abrasives and Ω the cross sectional area of jet.

The addition, term by term, of Eqs. (3) and (5), leads to

$$\dot{m}_a \frac{dV_a}{dx} + \dot{m}_w \frac{dV_w}{dx} = 0, \tag{8}$$

assuming that mass flow rates of both abrasive particle $\dot{m}_a$ and water $\dot{m}_w$ are held constant along the mixing and acceleration process, Eq. (8) becomes after simplification

$$d\,(\dot{m}_a\,V_a + \dot{m}_w\,V_w) = 0. \tag{9}$$

Integrating (9) leads to

$$\dot{m}_a\,V_a + \dot{m}_w\,V_w = R. \tag{10}$$

Eq. (10) expresses the constancy of momentum flux for the mixture, where R is a real positive constant completely determined by the initial conditions of jet formation.

For $x = 0$, we assume $V_a = V_{ao}$ and $V_w = V_{wo}$, so

$$R = \dot{m}_a\,V_{ao} + \dot{m}_w\,V_{wo}. \tag{11}$$

this also means that for distinct stations x_1 and x_2 one can write the following expression for the momentum flux

$$\dot{m}_a\,V_{a1} + \dot{m}_w\,V_{w1} = \dot{m}_a\,V_{a2} + \dot{m}_w\,V_{w2} = R. \tag{12}$$

The initial particle velocity V_{ao} inside mixing chamber is determined experimentally using appropriate instrumentation. The initial water velocity V_{wo} is estimated using Bernouilli's equation at the saphire nozzle exit, as proposed by Nadeau *et al.* (1991), as

$$V_{wo} = \sqrt{\frac{2\,P}{\rho_w}}. \tag{13}$$

The abrasive mass flow rate $\dot{m}_a$ is fixed experimentally while water mass flow rate $\dot{m}_w$ is given theoretically using the approximative following expression (Nadeau, *et al.*):

$$\dot{m}_w = 0.7\left(\frac{\pi\,D^2}{4}\right)V_{wo} \tag{14}$$

where the coefficient "0.7" expresses the vena contracta phenomenon which reduces the cross sectional area of a water jet as it leaves a nozzle.

2.5. Momentum equilibrium

According to the assumptions made in section 2.1, as momentum is conserved, it should be expected that for a sufficiently long mixing tube (large x-value), the particle and water velocities will tend towards the same limiting velocity V_{eq}. Eqs. (10) and (11), then give

$$V_{eq} = \frac{\dot{m}_a\,V_{ao} + \dot{m}_w\,V_{wo}}{\dot{m}_a + \dot{m}_w}. \tag{15}$$

assuming that mass flow rates of both abrasive particle $\dot{m}_a$ and water $\dot{m}_w$ are held constant along the mixing and acceleration process, Eq. (8) becomes after simplification

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2.6. Analytical resolution method

Eq. (1) for the particle acceleration and Eq. (10) for the mixture momentum flux may be rearranged as

$$\begin{cases} m_a\, V_a\, \dfrac{dV_a}{dx} = \dfrac{1}{2}\, \Omega_a\, \rho_w\, C_d\, (V_w - V_a)^2 \\ \dot{m}_a\, V_a + \dot{m}_w\, V_w = R. \end{cases} \tag{16}$$

The second equation of this system is amenable to the following variable changes

$$V_a = \frac{R}{\dot{m}_a} \cos^2\theta = \frac{R}{2\,\dot{m}_a}\,(1 + \cos 2\,\theta) \tag{17}$$

and

$$V_w = \frac{R}{\dot{m}_w} \sin^2\theta = \frac{R}{2\,\dot{m}_w}\,(1 - \cos 2\,\theta). \tag{18}$$

The derivatives of V_a and V_w with respect to x are,

$$\frac{dV_a}{dx} = -2\,\frac{R}{\dot{m}_a} \cos\theta \sin\theta\, \frac{d\theta}{dx} = -\,\frac{R}{\dot{m}_a} \sin 2\,\theta\, \frac{d\theta}{dx} \tag{19}$$

and

$$\frac{dV_w}{dx} = +2\,\frac{R}{\dot{m}_w} \cos\theta \sin\theta\, \frac{d\theta}{dx} = +\,\frac{R}{m_w} \sin 2\,\theta\, \frac{d\theta}{dx}\,. \tag{20}$$

The equation of motion for the particle, specified by the first equation of system (16), becomes:

$$\frac{dx}{d\theta} = -\,\frac{A_1}{B_1} \left[\frac{\sin 2\,\theta}{\left(\cos 2\,\theta - \frac{a}{b}\right)^2} + \frac{\sin 2\,\theta \cos 2\,\theta}{\left(\cos 2\,\theta - \frac{a}{b}\right)^2} \right] \tag{21}$$

where

$A_1 = \dfrac{m_a}{2\,\dot{m}_a^2}$	$B_1 = \dfrac{1}{4}\,\Omega_a\,\rho_w\,C_d\,b^2$	$a = \dfrac{1}{\dot{m}_w} - \dfrac{1}{\dot{m}_a}$	$b = \dfrac{1}{\dot{m}_w} + \dfrac{1}{\dot{m}_a}$

The analytical solution of Eq. (21) yields

$$x_k = \frac{A_1}{2\,B_1} \left[\ln \left| \cos 2\,\theta - \frac{\mathrm{a}}{\mathrm{b}} \right| - \frac{1 + \dfrac{a}{b}}{\cos 2\,\theta - \dfrac{a}{b}} + 2\,C \right]. \tag{22}$$

Eqs. (17) and (18) are rearranged as,

$$\cos 2\theta = 2\,\frac{\dot{m}_a}{R}\,V_a - 1 \tag{23}$$

and

$$\cos 2\theta = 1 - 2\,\frac{\dot{m}_w}{R}\,V_w \tag{24}$$

Substituting for cos 2 θ from Eqs. (23) and (24), above, we obtain:

– *Solid phase (particle):*

$$x_a = \frac{A_1}{2\,B_1}\left[\ln\left|2\,\frac{\dot{m}_a}{R}\,V_a - \frac{a}{b} - 1\right| - \frac{1+\frac{a}{b}}{2\,\frac{\dot{m}_a}{R}\,V_a - \frac{a}{b} - 1} + 2\,C\right] \tag{25}$$

– *Liquid phase (water):*

$$x_w = \frac{A_1}{2\,B_1}\left[\ln\left|2\,\frac{\dot{m}_w}{R}\,V_w + \frac{a}{b} - 1\right| + \frac{1+\frac{a}{b}}{2\,\frac{\dot{m}_w}{R}\,V_w + \frac{a}{b} - 1} + 2\,C\right]. \tag{26}$$

The integration constant C is given by the initial conditions of jet formation where $V_a = V_{ao}$ at $x_a = 0$, so we obtain

$$C = \frac{1}{2}\left[\frac{\frac{a}{b}+1}{2\,\frac{\dot{m}_a}{R}\,V_{ao} - \frac{a}{b} - 1} - \ln\left|2\,\frac{\dot{m}_a}{R}\,V_{ao} - \frac{a}{b} - 1\right|\right]. \tag{27}$$

For given conditions of jet formation, Eqs. (25) and (26) represent, respectively, the variations of abrasive particle velocity V_a and water velocity V_w as a function of distance x along the mixing tube, they are also called accelation models.

2.7. Interfacial virtual mass force

In this section, we assume that a particle is moving in the stream under the action of both drag and virtual mass force, the effect of which we add into Eq. (1). Following Lahey *et al.* (1980), a simplified expression of a virtual mass force per unit volume of particle, is given by

$$f_{vm} = \frac{1}{2}\,\rho_w\,V_a\,\frac{d\,(V_a - V_w)}{dx}. \tag{28}$$

Eq. (1) then becomes:

$$\rho_a\,V_a\,\frac{dV_a}{dx} = -f_d - f_{vm}. \tag{29}$$

A_a of Eq. 7 becomes equal to the addition of the drag and virtual mass forces per unit of distance along mixing tube. Eq. (10) still applies since the virtual mass force is interfacial. However, the system (16) becomes:

$$(30)\qquad \begin{cases} m_a\, V_a \dfrac{dV_a}{dx} = \dfrac{1}{2}\, \Omega_a\, \rho_w\, C_d\, (V_w - V_a)^2 + \dfrac{\mathrm{V}}{2}\, \rho_w\, V_a \dfrac{d\,(V_w - V)a}{dx} \\ \dot{m}_a\, V_a + \dot{m}_a\, V_w = R. \end{cases}$$

Proceeding as before, the equations corresponding to (25) and (26) are:

$$(31)\qquad x_a = \frac{A_1 + C_1}{2\,B_1}\left[\ln\left|2\,\frac{\dot{m}_a}{R}\,V_a - \frac{a}{b} - 1\right| - \frac{1 + \dfrac{a}{b}}{2\,\dfrac{\dot{m}_a}{R}\,V_a - \dfrac{a}{b} - 1} + 2\,C\right]$$

– *Liquid phase (water):*

$$(32)\qquad x_w = \frac{A_1 + C_1}{2\,B_1}\left[\ln\left|2\,\frac{\dot{m}_w}{R}\,V_w + \frac{a}{b} - 1\right| + \frac{1 + \dfrac{a}{b}}{2\,\dfrac{\dot{m}_w}{R}\,V_w - \dfrac{a}{b} - 1} + 2\,C\right]$$

where the integration constant C is given by relation (27), and with $C_1 = \dfrac{\mathrm{V}\rho_w}{4\,\dot{m}_a}\left(\dfrac{1}{\dot{m}_w} + \dfrac{1}{\dot{m}_a}\right)$. The Eqs. (31) and (32) represent, respectively, acceleration models for particle and water within the mixing tube under effects of both drag and virtual mass force.

2.8. Effect of air on acceleration process

As was mentioned previously, the Abrasive Water Jet system needs air to feed abrasive particles into the Jet. Experimental investigations (Tazibt, 1995) showed that air occupies nearly 95% of the total volume of the jet, so it becomes more than necessary to consider the effect of air on the acceleration process by including this effect in previous models.

The greater the volumetric proportion of air in the mixture, the lower is its density. Consequently, the interfacial forces, of viscous origin, that act on particles are also lowered, affecting the particle acceleration.

Added assumptions

Before considering the effect of air in the previous models, we propose to add some important assumptions which are listed below:

– the mixture is composed of two principal phases: *Particles/fluid,*

– the fluid phase is homogeneous and composed of droplets and air,

– the particle of fluid phase ρ_{fl} is given by the following mixing law:

$$(33)\qquad \rho_{fl} = \beta\rho_g + (1 - \beta)\,\rho_w.$$

We also assume that the momentum flux of the resulting mixture is held constant because the air has little influence since the contribution of each phase is proportional to its mass. This assumption makes it possible for us to substitute for the density of water ρ_w with that of the fluid phase ρ_{fl} in the equations of motion.

3. Discussion

3.1. Mixture without air

Considering only the effect of the interfacial drag force in the idealised case where the presence of air is neglected, the simulation of the acceleration modelling within the mixing tube yields two curves, Figure 2. The lower one shows that the particle velocity increases very quickly at the start of the acceleration process and tends towards equilibrium, while the upper curve shows that the water phase velocity is decreasing along the mixing tube. The equilibrium velocity V_{eq} of the two phases is derived using relation (15) and the initial water velocity V_{wo} is given by Bernouilli's Eq. (13).

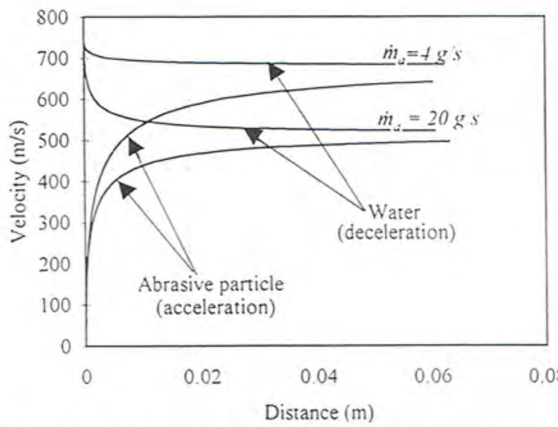

Fig. 2. – Estimated evolution of the particle and water velocities along the mixing tube, under the effect of drag force (*model 1*).

The equilibrium velocity, for $\dot{m}_a = 4\ g/s$ and $\dot{m}_w = 44.4\ g/s$, is $681.5\ m/s$ if the initial velocity of abrasives is assumed to be equal to $V_{ao} = 10\ m/s$. As one can see, the deceleration of the water phase is significantly affected by the amount of abrasive particles involved in the mixture. For $V_{wo} = 742\ m/s$, that deceleration increases from 7% to 28% when abrasive mass flow changes from $4\,g/s$ to $20\ g/s$. This result is consistent with that obtained by Nadeau *et al.* (1991).

From the comparison between *model 1* (effect of drag) and *model 2* (effect of both drag and virtual mass force), one can conclude that the virtual mass force decreases the abrasive particle velocity, Figure 3. One can also remark that for a distance of 6.8 cm, at impact with the workpiece, the difference between *model 1* and *model 2* is no longer significant. However, at a distance of 1 cm (in the zone of strong acceleration) from the initial mixing, the difference increases considerably.

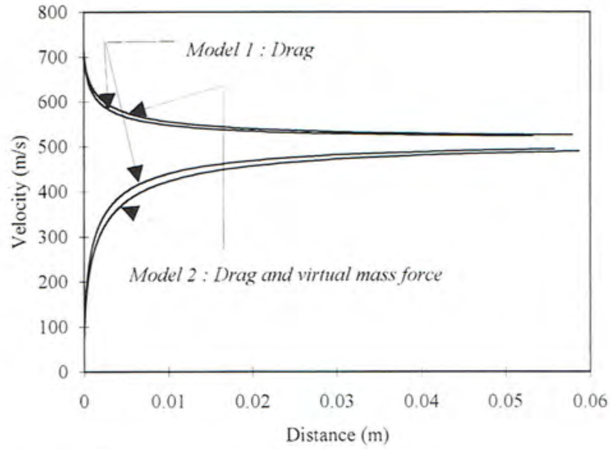

Fig. 3. – Comparison between *model 1* and *model 2* (drag and virtual mass forces effects).

Cutting power

Abrasive Water Jet Technology is used to cut materials, so cutting efficiency and control should be strongly affected by the kinetic power of particles at impact. That is to say, kinetic power is used for cutting so it becomes cutting power, ideally they are equal, following:

$$\left(\dot{E}_c = \frac{1}{2}\,\dot{m}_a\,V_{ac}^2\right) = (\dot{E}_r = \varepsilon\,\dot{m}_r) \tag{34}$$

Experimental work (Hashish, 1984; Nadeau *et al.*, 1991 and Tazibt, 1995) has shown that the removal mass flow rate $\dot{m}_r$ is a linear function of hydraulic water pressure P. According to relation (34), it is found that the particle kinetic power, which is estimated using the theoretical acceleration models, is also a linear function of water pressure, Figure 4. The same trends as those observed by Nadeau *et al.* (1991) are seen in

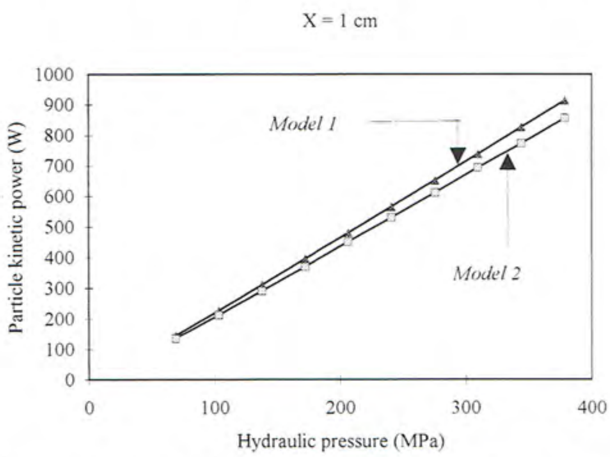

Fig. 4. – Estimated evolution of the particle kinetic power as function of hydraulic pressure.

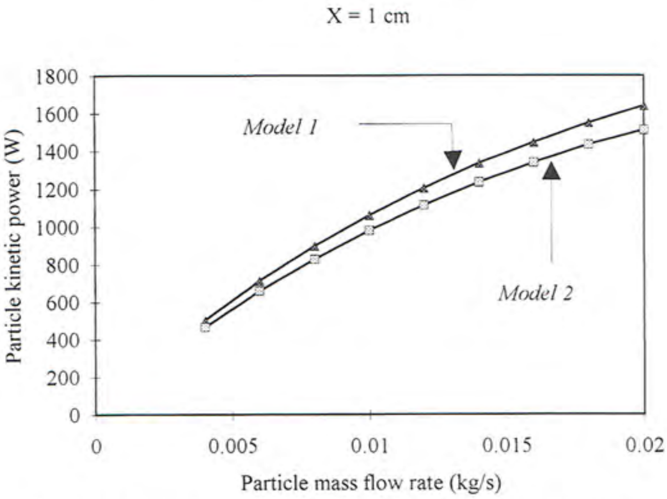

Fig. 5. – Estimated evolution of the particle kinetic power as function of the abrasive mass flow rate.

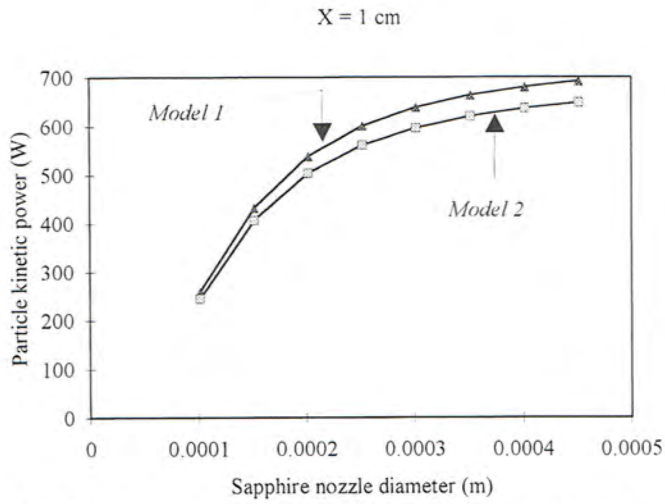

Fig. 6. – Estimated evolution of the particle kinetic power as function of sapphire nozzle diameter.

Figures 5 and 6, showing particle kinetic power, as function of particle mass flow rates and sapphire nozzle diameter respectively.

3.2 Mixture with air

The simulation of the acceleration modelling considering the real conditions of jet formation where air represents about 95% of the total volume of the jet (Tazibt, 1995), gives trends which are similar to those obtained in the idealised situation where the effects of air are neglected, Figure 7. As one can see in this figure, the abrasive particle velocity at impact is 60% of the equilibrium value. Figure 7 also shows the difference between the two situations. In the real case, the particle velocity is lower than as estimated in the idealised case while both the velocities remain less than the equilibrium value.

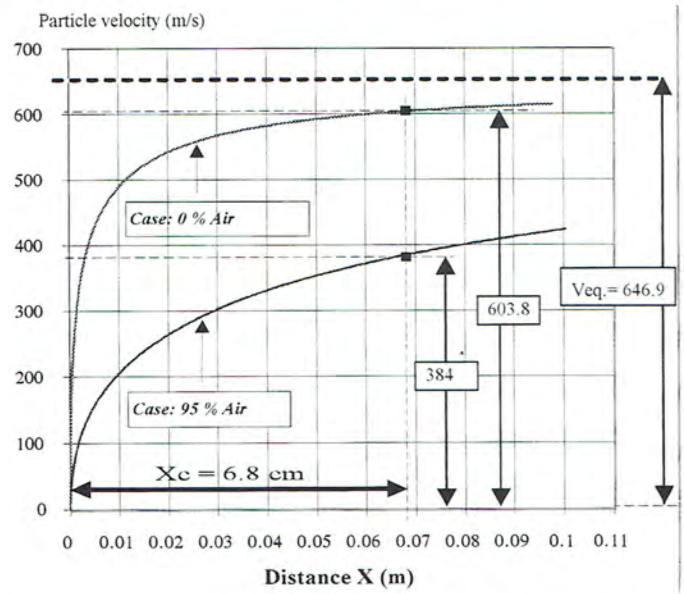

Fig. 7. – Estimated evolution of the particle velocity along the mixing tube in both idealised case (0% of air) and real one (95% of air), $P = 240$ MPa, $\dot{m}_a = 2.4$g/s, $D = 0.3$ mm, $D_a = 0.25$ mm, $\rho_a = 4140$ kg/m^3.

The influence of the amount of air on the abrasive particle velocity is shown in Figure 8. For a proportion of air greater than 70%, the particle velocity is not significantly affected by a decrease in the abrasive mass flow rate. However, in the same range of air proportion, the velocity of abrasives is lowered very quickly as the airflow increases.

In the real conditions of jet formation, the simulation of the velocity variation of each phase involved in the mixture, as function of a distance within the mixing tube, shows that the particle reaches its equilibrium velocity at a distance of impact of about 12 m

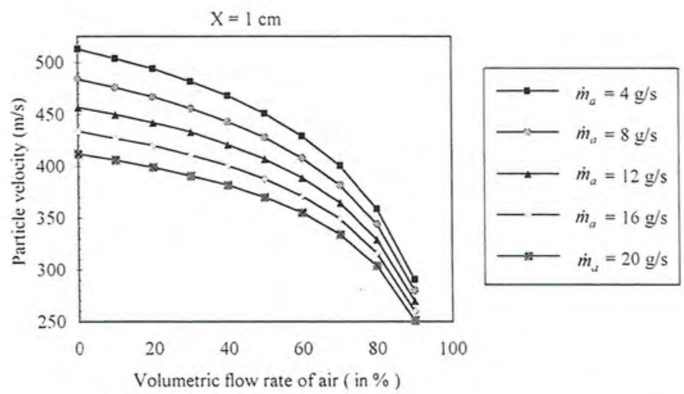

Fig. 8. – Simulation of the influence of air on the particle velocity.

within a mixing tube, Figure 9, while this distance is only around 12 cm when air effects are neglected. Thus while the conservation of momentum, with the present assumptions, leads to the particle velocity taking the same limiting value whether air is present or not, a much longer mixing tube will be required if there is a substantial volumetric flow of air. The lengths involved are greater than those used for the AWJ systems.

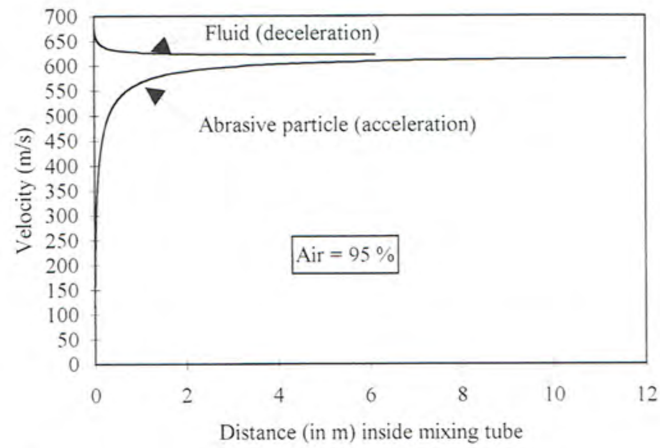

Fig. 9. – Simulation of the acceleration process for the real conditions of jet formation where air represents around 95% of the total volume of the jet, $P = 240\,\text{MPa}$, $\dot{m}_a = 4\,\text{g/s}$, $D = 0.3\,\text{mm}$, $D_a = 0.282\,\text{mm}$, $\rho_a = 4140\,\text{kg/m}^3$.

The comparison of the theoretical results with experimental ones is a complex task because of the lack of an appropriate experimental device for the measurement of particle velocity inside a mixing tube (Tazibt, 1995). However, Miller (1991) has used an electronic velocimeter to measure particle velocity inside a mixing tube whose diameter and length were, respectively, 14 mm and about 4 m. The particles where spheres of 3.26 mm diameter, the nozzle orifice diameter was 2 mm and air represented nearly 98% of the total volume of the jet.

As can be seen, the experimental parameters used by the author differed by a factor of about ten from those used in cutting materials. In Figure 10, one can notice that for

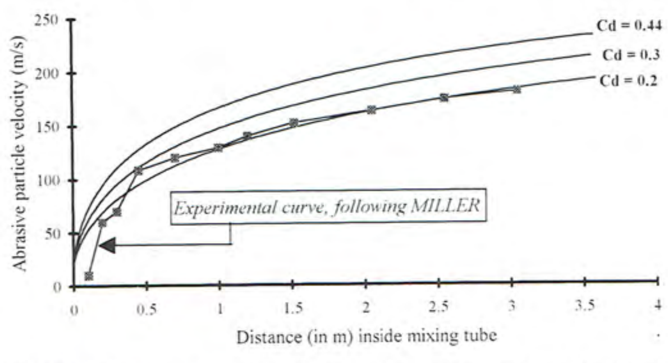

Fig. 10. – Comparison between the theoretical evolution of particle velocity with experimental results obtained in Miller (1991), $\rho_a = 7840\,\text{kg/m}^3$.

drag coefficient $C_d = 0.2$, a good agreement is recorded between theoretical estimation of particle velocity and experimental one, in the same conditions listed above.

4. Validation by an experimental correlation

The contention that the rate of the material removal is directly related to the kinetic power of the particles is supported by experimental work (Tazibt, 1995). The data are correlated by an expression of the form

$$H = \frac{1}{0.6\,\rho\,du}\,[\mathrm{A}\,V_{ac}^2 + B] \tag{35}$$

in which the 0.6 is a shape coefficient for the shallow slit, ρ is the density of the material being cut and A and B are functions of the working parameters, given in Tazibt (1995); Figure 11 shows curves deduced from Eq. (35). The values are near a collection of data obtained cutting ductile steel specimens (Tazibt, 1995). The accuracy of the correlation falls off at high abrasive mass flow rates, above the optimum of about 250 g/mn for the case shown in the figure. This is a result of the kinetic energy of some particles being lost in collisions with cut material or other spent particle.

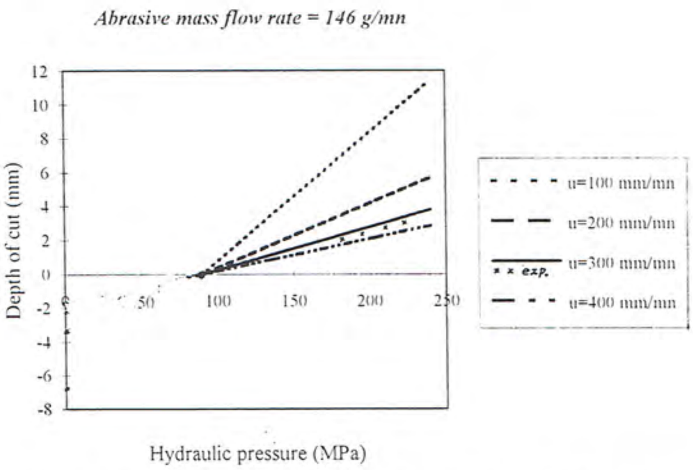

Fig. 11. – Estimated evolution of the depth of cut as function of hydraulic pressure, $D_a = 0.25$ mm, $D = 0.3$ mm, $\rho_a = 4140$ kg/m^3, $X = 6.8$ cm.

5. Conclusion

We have shown that an analytical solution for particle motion is possible. We have considered the effects of drag and virtual mass force in a high speed abrasive water jet. The analytical solutions are original and they describe the expected evolution of velocity of each phase (solid and liquid) along the mixing tube.

Their simplicity makes both the theoretical study of the influence of the working parameters of the acceleration process and an examination of effects of air flow on particle velocity possible. The present investigations are consistent with earlier experimental and theoretical work (Hashsish, 1984; Miller and Archibald, 1991 and Nadeau *et al.,* 1991). The principle results are summarised below:

– For an air volumetric flow rate greater than 70%, the acceleration process is independent of the abrasive mass flow rate. However, for this range of air flow the particle velocity is lowered considerably while it should be sufficient to consider only the effect of the drag force on the particle.

– For an air volumetric flow rate of 95%, the velocity of particle arriving at the workpiece is about 60% of its equilibrium value in a cutter with the dimensions considered here.

– Air flow greatly increases the distance at which a particle reaches its equilibrium velocity: for no air, this is reached near $X = 12\,\mathrm{cm}$ while for 95% air, a particle reaches the same equilibrium velocity but at a distance of about $X = 12$ metres.

The experimental validation of the present accceleration modelling is detailed in Tazibt (1995). It is conducted using an experimental correlation which links the theoretical particle velocity with the depth of cut, measured experimentally.

The theoretical modelling of the acceleration process coupled with the experimental correlation provides good estimation of depth of cut and traverse rate in the linear zone of cutting and for abrasive mass flow rates which are less than the optimum. However, for abrasive mass flow rates greater than the optimum, a deviation between theoretical estimation of depth of cut and experiment is observed. This situation can be explained by the collision of abrasive particles as a result of which part of their kinetic power is wasted. It is therefore suggested that materials be cut with abrasive mass flow rates which are less than optimum.

The physical assumptions underlying this model are very simple, so that by correlating more detailed information, it should be possible to increase the accuracy of prediction for the particle acceleration.

Finally, we can conclude that the present study contributes towards the solution of some general acceleration problems encountered in high speed two-phase flow (solid/fluid), and helps users of AWJ cutting to predict either the depth of cut or the traverse rate for given conditions of jet formation and operation parameters.

– Fig. 2: $P = 275\,\mathrm{MPa}$; $D_a = 0.3\,\mathrm{mm}$.
– Fig. 3: $P = 275\,\mathrm{MPa}$; $\dot{m}_a = 20\,\mathrm{g/s}$.
– Fig. 4: $\dot{m}_a = 4\,\mathrm{g/s}$; $D_a = 0.2\,\mathrm{mm}$.
– Fig. 5: $P = 240\,\mathrm{MPa}$; $D_a = 0.3\,\mathrm{mm}$.
– Fig. 6: $P = 275\,\mathrm{MPa}$; $D_a = 0.2\,\mathrm{mm}$.
– Fig. 10: $P = 69\,\mathrm{MPa}$; $D_a = 3.26\,\mathrm{mm}$; $\dot{m}_a = 104\,\mathrm{g/s}$; $\dot{m}_w = 1100\,\mathrm{g/s}$.
Simulations are conducted with $V_{ao} = 10\,\mathrm{m/s}$ (Fig. 10: $20\,\mathrm{m/s}$), nozzle diameter = $0.33\,\mathrm{mm}$ (Fig. 10: 2 mm).

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