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Abstract

In this paper, a general modelling for the acceleration process of abrasive particles in a high pressure water jet is presented. For this purpose, a new mathematical method is used which consists of the analytical resolution of the differential and non-linear equation of particle motion within a high speed waterjet flowing in a mixing nozzle (mixing tube). As a result, the equations of motion reveal that the particle velocity increases while the fluid jet velocity decreases as function of the distance within the mixing tube with respect to the assumption of conservation of momentum along the acceleration process.

The original method of resolution used here enabled the authors to take account of all of the interfacial forces that act on the particle such as drag and virtual mass force. Moreover, it becomes more possible to predict the particle velocity at impact considering the real conditions of jet formation where air is entrained into the jet to feed abrasive particles by the Ventury phenomenon.

The experimental validation of the present theoretical modelling has been conducted successfully by means of an experimental correlation which links the estimated particle velocity at impact and the experimental depth of cut. So the theoretical modelling of particle acceleration coupled with the experimental correlation provides a good estimate for control of cutting parameters such as hydraulic pressure of water, abrasive mass flow rate, traverse rate and depth of cut.

1. Introduction

Cutting with high velocity abrasive water jets is now widely used in industrial applications because of its ability to cut several materials without modifying their mechanical properties. This aspect is more than required for cutting such materials as plastics, leather and composites, so the hardest materials can be cut more economically than by using conventional cutting systems.

There are two types of abrasive water jets. The first one is called the Abrasive Slurry Jet (ASJ) where abrasives are premixed with water in a high pressure tank. The resulting mixture is directly pumped and accelerated within a long high pressure pipe before arriving at the workpiece. The particularity of such a system is that the mixture is composed of two phases which include abrasive particles and water.

The second type is called the Abrasive Water

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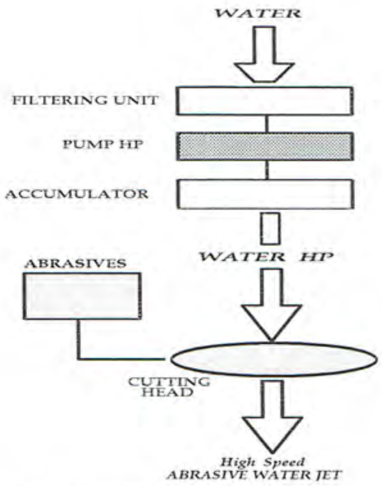

Fig. 1. Abrasive Water Jet formation.

Jet (AWJ) or PASER and uses an entrainment system to feed abrasives. In the AWJ systems, abrasives are sucked into the jet by the Ventury phenomenon from a hopper to the mixing chamber through in the flexible. The mixture is composed of abrasive particles and water and air. However a particle is accelerated by the fluid jet (droplet and air) within a short carbide mixing tube before arriving at the workpiece which is also called the cut material.

In this study, the focus is on the Abrasive Water Jet using an entrainment system in which low pressure water is filtered and raised to high pressures by means of a pump which is built from a standard Flow Systems Inc. The components of such a system included an intensifier and an accumulator which provides a constant high pressure water supply to the nozzle from which waterjet exits with a high velocity. The range of hydraulic water pressure is from 140 to 400 MPa and the velocity of water at the nozzle exit is about 400 to 900 m/s (see Fig. 1).

The use of this cutting process poses the problems of the control and optimization for working parameters such as hydraulic pressure, mass flow rate of abrasives, nozzle diameter, velocity of abrasive particles and, especially, practical parameters such as traverse rate and depth of cut. The complexity of the cutting mechanism makes it difficult to model theoretically taking account of the great number of parameters involved. Actually, this difficulty is circumvented by decoupling the process, considering the acceleration and cutting aspects separately. These two processes must be coupled together through the velocity of particles at impact.

A knowledge of the particle velocity is of great importance because, due to the kinetic energy of each abrasive particle, the material is cut by wear [1,3,5]. In this spirit, Hashish [3] has developed a cutting model in which the depth of cut depends on the velocity of the abrasive particle. In his model, the velocity of the particle is estimated by means of a conservation law of momentum at equilibrium; it corresponds to that velocity that the particle would have at impact if the mixing tube were sufficiently long [3,5]. In practice, that situation is not attainable, so the present theoretical analysis of the particle acceleration process has been developed in order to estimate the velocity of the abrasive particle at practical distances of impact which are usually used in systems for cutting with AWJ.

In the following sections, we present the theoretical analysis and the experimental method of validation.

2. Review of existing modelling

The most important existing modellings of the acceleration process encountered in the literature, are those of Drew [2], Abudaka [1] and Nadeau [5]. Drew [2] has presented more complete equations of momentum for the phases involved in the mixture which is flowing within a nozzle, but the analytical resolution of such equations is impossible because they are differential and non-linear.

Abudaka [1] has presented a simple equation of motion of a particle within the jet in the mixing tube under the action of the interfacial drag force. However, he considered that the fluid jet velocity is held constant during the acceleration process. In addition, his modelling has two constants which are determined in an ad hoc fashion.

Using the approximative method of Runge-Kutta, Nadeau [5] has resolved numerically the differential and non-linear equation of motion of a particle within the water jet in the mixing tube. As a result, he has plotted the acceleration of particles and the deceleration of water phase as function of distance within the mixing tube. While the results are more interesting, the numerical solution method considered only the effect of the interfacial drag force and neglected the effect of air on the particle velocity.

We can conclude, finally, that there is no simple and analytical solution of the differential and non-linear equation of motion of a particle which describes mechanical phenomena as a whole equation of momentum.

3. Hypothesis of study

In the first approach, we consider the following assumptions:

(1) the mixture is composed of two phases — particles and water;
(2) the particle is isolated and accelerated by water;
(3) momentum is constant along the mixing tube;
(4) momentum is transferred from water to the abrasive particles;
(5) the effect of air is neglected;
(6) friction and gravitational forces are neglected;
(7) the abrasive velocity is held constant between the mixing tube exit and the workpiece.

4. Mechanism of mixing and acceleration process

As shown in Fig. 2, high pressure water enters the mixing chamber with a high speed (about 600 to 900 m/s) through a nozzle of 0.05 to 0.5 mm diameter and creates a depression. The abrasive particle is entrained by air (Ventury phenomenon) within a flexible from a hopper and enters the mixing chamber with a low velocity. As the resulting mixture moves down the mixing tube

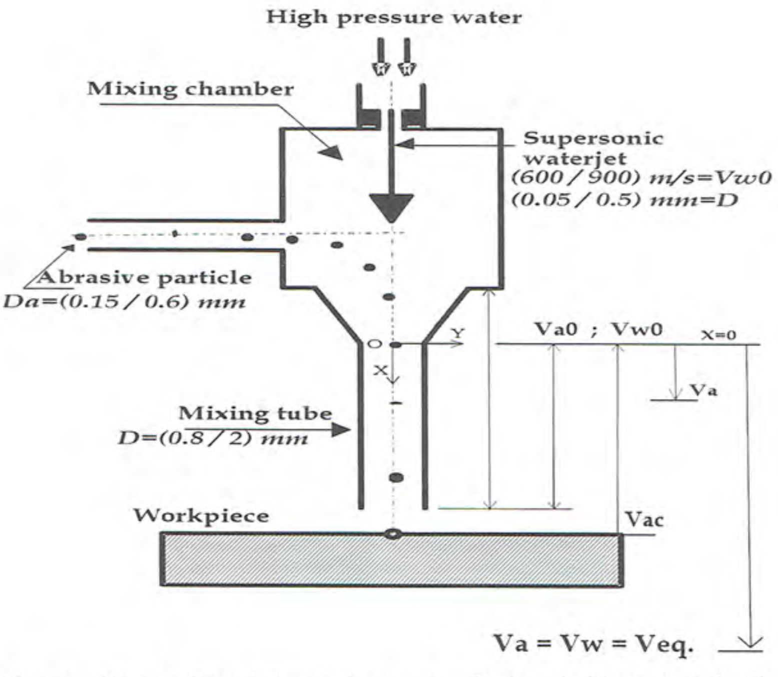

Fig. 2. Schema of Abrasive Water Jet cutting head and mechanism of mixing and acceleration process.

of 0.8 to 2 mm diameter, momentum is transferred from the high velocity of water to the low velocity of the abrasive particle.

According to the conservation of momentum, the particle velocity increases while the liquid phase one decreases along the mixing process. For a sufficiently long mixing tube, the abrasive particle and water velocities will come to be equal. However, knowledge of the limiting value of the abrasive particle velocity is of a great importance for design and operation [3].

5. Mathematical description of motion

The mixture can be represented by two separated phases: liquid (water) phase and solid (particle) phase. These phases interact at the interface where interfacial forces are developed along the flow. The previous mathematical description of motion and the assumptions above allowed us to write the equations of momentum as follows [6]:

5.1. Solid phase (particle)

$$\alpha\rho_a V_a \frac{dV_a}{dx} = -\alpha\frac{dP}{dx} + M_a. \tag{1}$$

5.2. Liquid phase (water)

$$(1-\alpha)\rho_w V_w \frac{dV_w}{dx} = -(1-\alpha)\frac{dP}{dx} + M_w. \tag{2}$$

M_a and M_w are given by:

$$M_w = -M_a = \alpha(F_d + F_{vm}), \tag{3}$$

where

$$F_d = \frac{3}{4}C_d\frac{\rho_w}{D_a}(V_a - V_w)\,|V_a - V_w| \tag{4}$$

and

$$F_{vm} = \frac{1}{2}\rho_w V_a \frac{d(V_a - V_w)}{dx}. \tag{5}$$

6. Equation of motion of the particle

Assuming that the solid volumetric fraction $\alpha \ll 1$ and after simplification, Eqs. (1) and (2) lead to

$$\rho_a V_a \frac{dV_a}{dx} = -F_a - F_{vm} + \rho_w V_w \frac{dV_w}{dx}, \tag{6}$$

which is the differential equation of an isolated abrasive particle motion on the axis of the mixing tube. Eq. (6) is non-linear and has two unknowns, V_a and V_w. To solve this equation, a second equation is required which is based on the conservation of momentum, of the mixture.

7. Momentum of the mixture

Following Ref. [6], the addition, member to member, of the two equations of momentum transfer term at the interface results, after the simplification, as:

$$m_{1a}V_a + m_{1w}V_w = m_{1a}V_{a0} + m_{1w}V_{w0} = R. \tag{7}$$

Eq. (7) translates the conservation of momentum (per unit time) according to the hypothesis made in Section 3. Notice that the right-hand side of (7), R, is given by the formation conditions of the abrasive water jet so it is completly determined.

For a sufficiently long mixing tube, the abrasive particle and the water phase velocities tend towards the limit V_{eq}, as follows:

$$V_{eq} = \frac{m_{1a}V_{a0} + m_{1w}V_{w0}}{m_{1a} + m_{1w}}. \tag{8}$$

8. Accelaration modelling

Recalling that such a modelling is an equation giving the velocity evolution along the mixing tube for given conditions of jet formation, the analytical method detailed in Ref. [6] offers the possibility to take account all forces which participate to the particle motion. These forces are in

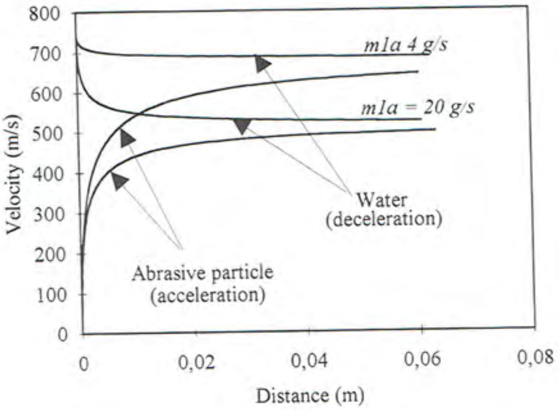

Fig. 3. Estimated evolution of particle and water velocities along mixing tube, under effect of drag force (Model 1).

particular interfacial drag, virtual mass, mechanical diffusion, friction and gravitational forces. It then becomes possible to set a simple mathematical equation which includes the effects of these forces. The effect of air entrainment can also be considered [6].

If the effect of air is neglected, the most important forces that act on the abrasive particle are the interfacial drag and the virtual mass force [6], so the acceleration models are presented for these two forces.

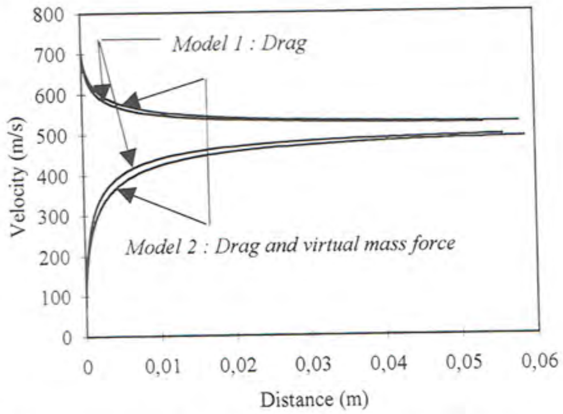

Fig. 4. Comparison between Model 1 and Model 2 (drag and virtual mass forces effects).

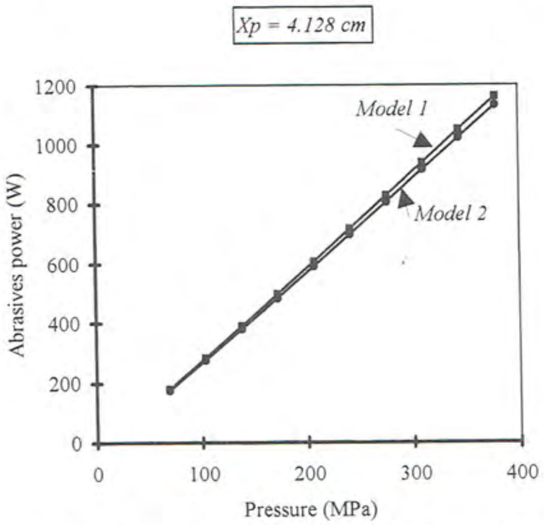

Fig. 5. Estimated evolution of kinetic power of particles as a function of hydraulic pressure.

8.1. Interfacial drag (Model 1)

(a) solid phase (abrasive particle)

$$x_{\mathrm{a}} = \frac{A_1}{2B_1}\left[\ln|2qV_{\mathrm{a}} - s| - \frac{s}{2qV_{\mathrm{a}} - s} + 2C\right]; \quad (9a)$$

(b) liquid phase (water)

$$x_{\mathrm{w}} = \frac{A_1}{2B_1}\left[\ln|h - 2pV_{\mathrm{w}}| - \frac{s}{h - 2pV_{\mathrm{w}}} + 2C\right]. \quad (9b)$$

8.2. Drag and virtual mass force (Model 2)

(a) solid phase (abrasive particle)

$$x_{\mathrm{a}} = \frac{A_1 + C_1}{2B_1}\left[\ln|2qV_{\mathrm{a}} - s| - \frac{s}{2qV_{\mathrm{a}} - s} + 2C\right]; \quad (9c)$$

(b) liquid phase (water)

$$x_{\mathrm{w}} = \frac{A_1 + C_1}{2B_1}\left[\ln|h - 2pV_{\mathrm{w}}| - \frac{s}{h - 2pV_{\mathrm{w}}} + 2C\right] \quad (9d)$$

where the integration constant, C, is given by the conditions at the entrance of the mixing chamber according to

$$C=\frac{1}{2}\left[\frac{s}{2qV_{\mathrm{a0}}-s}-\ln|2qV_{\mathrm{a0}}-s|\right] \tag{9e}$$

and with

$$s=1+\frac{a}{b},\qquad a=\frac{1}{m_{\mathrm{1w}}}-\frac{1}{m_{\mathrm{1a}}},\qquad A_1=\frac{M}{2m_{\mathrm{1a}}^2},$$

$$C_1=\frac{Vb\rho_{\mathrm{w}}}{4m_{\mathrm{1a}}},\qquad p=\frac{m_{\mathrm{1w}}}{R},$$

$$h=2-s,\qquad b=\frac{1}{m_{\mathrm{1w}}}+\frac{1}{m_{\mathrm{1a}}},\qquad B_1=\frac{Kb^2}{4},$$

$$K=\tfrac{1}{2}\Omega_{\mathrm{a}}\rho_{\mathrm{w}}C_{\mathrm{d}},\qquad q=\frac{m_{\mathrm{1a}}}{R}.$$

9. Effect of air on the particle velocity

The Abrasive Water Jet technology using the entrainment system needs air to feed abrasives into the jet. Experimental investigations showed that the most important proportion of jet volume is occupied by air, so we have added some assumptions in order to study the influence of this parameter on the abrasive particle velocity.

9.1. Some added assumptions

(1) The mixture is composed of two principal phases — particles and fluid;
(2) the fluid phase is homogeneous and consists of droplets and air;
(3) the resulting mixture is homogeneous;
(4) the particle is isolated and accelerated by fluid flow.

9.2. Contribution of air

Air makes the fluid jet density low but increases the mixing process between water and particles.

According to the added assumptions above, a mathematical translation of the contribution of air in the acceleration process is more possible on the principle of substituting water density by that of the fluid in the previous equations of acceleration modelling.

The fluid jet density is given by:

$$\rho_{\mathrm{fl}}=\beta\rho_{\mathrm{air}}+(1-\beta)\rho_{\mathrm{w}} \tag{10}$$

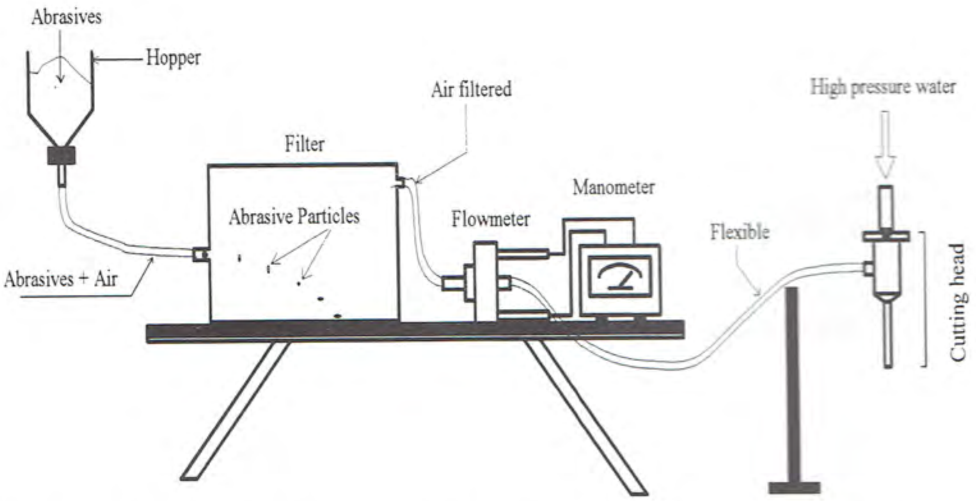

Fig. 6. Partial experimental device allowing for measurements of air volume in the jet and mean velocity of both particles and air at the entrance of the mixing chamber.